

Provided for non-commercial research and educational use only.
Not for reproduction or distribution or commercial use.



This article was published in an Elsevier journal. The attached copy is furnished to the author for non-commercial research and education use, including for instruction at the author's institution, sharing with colleagues and providing to institution administration.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>

Geographic variation and parameter assessment in generalized algebraic difference site index modelling

Andrés Bravo-Oviedo^{*}, Miren del Río, Gregorio Montero

Departamento de Sistemas y Recursos Forestales – Department Forest Systems and Resources, CIFOR-INIA, Ctra. A Coruña, km 7.5, 28040 Madrid, Spain

Received 27 November 2006; received in revised form 10 April 2007; accepted 12 April 2007

Abstract

Parameter assessment in site index modelling has proved useful prior to model specification. A base-age invariant model that supports polymorphism and variable asymptotes was fitted for Mediterranean Maritime pine in the inland part of Iberian Peninsula. A generalized algebraic formulation of Hossfeld's model was found to be the most appropriate model in terms of biological consistency and fit statistics throughout the range of the data. Models were fitted to data from stem analysis and permanent sample plot measurement using a dummy variable approach and a continuous-time autoregressive structure to correct for autocorrelation. Model performance was evaluated using Jack-knife techniques applied to each growth series. The selected model was used in a regional-based comparison of growth patterns. The parameter identified as 'global' in the first part of the study was expanded with dummy variables to assess the differences among regions. The stands located in the Northern Plateau, the Central Mountain Range, and South Eastern Mountains, exhibited different growth patterns from one another. However, these regions show a similar growth pattern to stands located in their origin of dispersion (Iberian Mountain Range). Consideration was also given to other factors which might lead to variation, such as environmental conditions.

© 2007 Elsevier B.V. All rights reserved.

Keywords: Regional growth pattern; Parameter assessment; *Pinus pinaster*

1. Introduction

Site index modelling techniques have undergone a major evolution from the first hand drawn site index curves to the empirical and biological based equations. Advances in computing and understanding of growth processes have in turn promoted site index modelling as an area of research within forest science.

In early forest site productivity studies, the main objective of site index modelling was to determine the potential stand productivity when growth and yield were the main concern of forest research. More recently, the identification of additional objectives such as multi-functionality, conservation or biomass assessment, has brought demands from forest managers for better models which improve our understanding of the processes involved in forest ecosystems. Height growth and site index are aspects which need to be addressed within this context.

Height development properties should be represented by a height growth model (in the form of a non-decreasing curve) which accounts for polymorphism, point of inflection and asymptote. Bailey and Clutter (1974) introduced a technique now known as the algebraic difference approach (ADA) for modelling site index curves. This approach accounts for all the properties at the same time and involves making one parameter site-dependent in order to achieve a base-age invariant polymorphic model with a common asymptote. Cieszewski and Bailey (2000) generalized the ADA formulation by means of making 'a priori' assumptions about the relationship of some growth function parameters with a theoretical growth intensity factor (GIF), which is considered to be a variable or a function of variables representing site (climate, soil attributes) and genetic factors.

This methodology allows both polymorphism and variable asymptotes to be described at the same time where parameters identified as site-dependent account for such properties. The parameters chosen to be related to the growth intensity factor must lead to solvable equations in a closed-form (Cieszewski, 2004). This implies that some combinations of parameter site dependency are not possible. Therefore, a key aspect of this

^{*} Corresponding author. Tel.: +34 913 473 578; fax: +34 913 476 767.
E-mail address: bravo@inia.es (A. Bravo-Oviedo).

method is the identification of site parameters. However, little consideration has been given to determining the sort of relationships that exist between the parameters and the GIF. The main reason for this is the ‘unobservability’ of such factors because of the concurrence of many factors, which in practical terms is difficult to measure. Despite this, Cieszewski and Bailey (2000) assume that small values for GIF represent low growth intensity and high values represent high growth intensity. Thus, low growth might be considered comparable to low production and high growth to high production, as production is the sum of different growth periods. Site-parameter selection is commonly conducted according to growth theory or hypothetical assumptions, but observed site parameter relationships can also be used to define explicit dynamic growth equations. Therefore, the relationship between the potential stand production, usually measured as site index, and the parameter estimates of base equations, could provide a valuable insight into the development of the growth intensity factor.

Maritime Pine (*Pinus pinaster* Ait.) is one of the most important forest species in South and South-western Europe, covering more than four million hectares (Ribeiro et al., 2001), in Spain, Portugal, France and Italy. The great variability within the species and its widespread distribution allows the existence of, at least, two groups or races: the Atlantic Maritime Pine (AMP), which is the more productive of the two, and the Mediterranean Maritime Pine (MMP) which is less significant in terms of wood production but which plays an important role in resin production, restoration programmes and protection from erosion, as well as in the maintenance of genetic biodiversity. Its role as a species for plantation and restoration programs has been highlighted in provenances tests (Danjon, 1994; Alía et al., 1997) and genetic variability studies (Vendramin et al., 1998; Salvador et al., 2000; González-Martínez et al., 2001).

Traditionally, MMP in the south-western Mediterranean area has been used for resin production and soil protection against mobile continental dunes. Today, the MMP resin market has shrunk as the more competitive petroleum-derived products have taken over. The market has also been affected by exports from emerging economies. Management plans for thousands of hectares that had been tapped for decades have been abandoned resulting in the appearance of in-growth and advanced regeneration. It is clear that objectives of the management plans for these stands need to be modified to include biodiversity, wood or biomass production as the main priorities rather than resin production. An important step towards these new objectives involves the assessment of potential site productivity.

The first attempt at modelling dominant height growth of non-tapped MMP in south-western stands was carried out by Pita (1967) using a quadratic modification of the equation proposed by Hossfeld (1822), fitted according to the natural curves method. Bravo-Oviedo et al. (2004) published a set of site index curves calculated using algebraic differences through Bailey and Clutter’s (1974) modification of Schumacher’s (1939) model. However, the use of these curves leads to biased

behaviour, especially for older and better quality trees for which there were insufficient data. As a result, overestimation of site quality in young stands occurs. This problem often arises in site index modelling because the better quality trees are harvested, planted, or naturally regenerated in preference to the poorer ones. Despite the expense and occasional problems caused by insufficient data, perhaps the best way of correcting site quality overestimation in younger stands is to collect new stem analysis data in order to fit a new model.

The interaction of genetic systems and environmental factors leads to the development of geographic variations in the species (Morgenstern, 1996), with variable growth patterns. Salvador et al. (2000) defined three regional groups of genetic variation for Mediterranean Pine, according to the geographic structuration of the species. These regions correspond to the hypothetical origin of dispersion of the species after glaciations, from the East of the Iberian Peninsula to the West and the North. Ecological differences affecting tree growth were detected for the species by Nicolás and Gandullo (1967). In their study, they made a first attempt at ecotype division (according to soil and climatic conditions) for the all the populations of both AMP and MMP distributed throughout Spain. The distribution of the species within the Iberian Peninsula is quite fragmented which results in isolated stands often displaying different growth patterns.

The poor performance of existing models, the interregional variability and the flexibility showed by generalized algebraic difference approach are the main factors which have prompted this paper. The aim of this study is to develop a well behaved set of site index curves (using the GADA formulation), and to investigate the relationships between parameter and growth intensity factor during the early stages of model development to explicitly define dynamic models.

2. Material and methods

2.1. Data

In 1964, a large productivity experiment, consisting of a permanent sample plot network, was established covering the distribution of the principal forest species in Spain (Montero et al., 2004). The Mediterranean *Pinus pinaster* (MPP) trial consisted of 96 plots in natural and sown stands, covering different site qualities, ages and densities according to Forestry Commission recommendations (Hummel et al., 1959). Next to each plot and in similar stand conditions, two or three dominant trees were cut when the plots were established, resulting in 281 stem analyses (1966DB). Stand ages in the database ranged between 18 and 127 years. Younger trees were over-represented at the better sites. Therefore, stem analysis was performed for trees older than 70 years felled in 2005 (2005DB). Two sample trees were selected in 36 locations according to Assmann’s dominance criterion (Assmann, 1971), and sectioned at the base, at a height of 1.3 m and every meter thereafter. Microsite variations were assumed to be represented by averaging dominant trees in each location. The database was completed by combining the stem analysis data with records from

Table 1
Summary of the data used in the study by regions and combined^a

Region	No. stands	Age			Dominant height		
		Average	Min	Max	Average	Min	Max
NPS	30	64 (19.89)	33	106	15.02 (2.77)	9.21	20.49
CMS	39	68 (26.67)	25	140	17.77 (4.39)	8.3	28.25
IMS	31	93 (33.25)	41	170	13.93 (3.26)	7.6	20.07
SES	23	100 (31.62)	61	180	17.01 (2.99)	9.02	21.67
TOTAL	123	80 (32.91)	25	180	15.80 (3.90)	7.6	28.25

^a In parenthesis standard deviation.

permanent sample plots at the same location where possible, as long as the growth trend was similar. ‘Similar’ in this instance, means that the growth trajectory should be continuous and without sudden changes in the growth pattern, which otherwise would indicate measurement errors. Some observations were deleted, skipping to the following measurement interval in the growth series as long as the growth trend did not appear to differ. A combination of data from stem analysis and permanent sample plots was also used in site index modelling by García (2005) and Diéguez-Aranda et al. (2005b). Table 1 shows the data used in the analysis, including their main descriptors.

Alfía et al. (1996) defined 21 seed provenances based on geographic, climatic and edaphic features of *Pinus pinaster* stands. The data for our study come from nine of these provenances, distributed in four biogeoclimatic regions according to Elena-Rosselló et al. (1997). Having considered the aforementioned studies and recognizing that stand continuity is important in management planning, we defined four groups for comparing MMP. The groups are labelled according to their location. The Northern Plateau Stands (NPS) are characterised by sandy soils (although clay horizons are also common along the basin of the Duero River), mean annual temperature of 11.3 °C and annual precipitation of 486.4 mm. The quality of these stands is considered to be low and stems are often poorly formed. The Central Mountain Range Stands (CMS) are located in the central part of the Iberian Peninsula; a wide area covering three seed provenances, all on granite and gneiss soils, at altitudes above 1000 m, annual precipitation of 635 mm and a mean annual temperature of 12.5 °C. The Iberian Mountain Range Stands (IMS) are located in the Eastern part of Spain on sandstone soils (Bundtsandstein facies), where the mean annual temperature is 10.5 °C and annual precipitation reaches 546.7 mm. Finally, the Segura-Alcaraz Mountain Stands (SES), are located in the south-eastern part of the Iberian Peninsula on soils derived from dolomites with an abundance of clay and silt. The mean annual temperature for the region is 13.1 °C and annual precipitation is 651.7 mm. Fig. 1 shows the groups and the species distribution.

2.2. Modelling approach

Cieszewski (2004) presented 80 GADA based, true base-age invariant or dynamic equations using three assumptions which affect the relationship between parameters and GIF: linear, inverse-saturation and quadratic. The GADA consists of

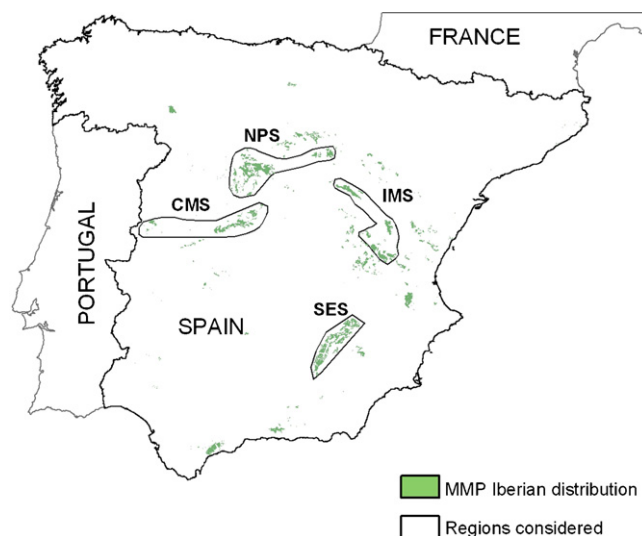


Fig. 1. Mediterranean maritime pine distribution in the Iberian Peninsula and geographic areas considered.

selecting a base equation, identifying the parameters that are related to the X variable and determining their functional relationship. Once the model is explicitly defined, the equation is solved for X and the variables H and t are substituted for their initial conditions H_0 and t_0 . The selection of the parameters related to the X variable and the relationship between them should permit a closed form solution (Cieszewski, 2004).

The potential relationship between the parameters of the base functions tested and the observed site index found in each growth series were checked, assuming that the observed site index could account for much (but not all) of the unknown variation in the X variable. First, the actual site index, measured as the true height of the growth series at age 70, was determined; then the base equations were fitted to each growth series and finally, a regression analysis of parameters on actual site index was carried out. A graphical analysis of parameters on actual site index was also performed to detect the type of relationship (linear, nonlinear, inverse, quadratic ...) and to facilitate the identification of site specific parameters as well as the most likely expression for the surrogate of the growth intensity factor. The selection of the base age used to estimate the actual site index may be arbitrary since the height of unsuppressed dominant trees, which have been dominant throughout their life, is the result of the conjunction of all growing factors regardless of the age considered. In order to corroborate this assumption we plotted site index with differing base ages versus parameter estimates.

Cieszewski (2000) described two different fitting methods to obtain unbiased base age invariant equations: the consecutive reiterations approach (CRA), first proposed by Tait et al. (1988) and the dummy variable approach (DVA). The general idea of both methods is to estimate simultaneously the global and the specific parameters, regardless of the height adopted for any base age. In the CRA method, the first step is to estimate the global parameters, then these are considered as constants and specific parameter are estimated, finally the site-specific estimated value is used as an observed value and global

parameters are refit until stabilize. The DVA uses an initial height as a starting value for the site-specific parameter for each growth series, assuming all measurements of a particular growth series to have the same starting value. The dynamic model takes the following form:

$$y = f \left[\sum_{i=1}^n \alpha_{si} d_i; \beta; t_j; t_0 \right] + e_{ij} \quad (2)$$

where y is any growth variable, in this case dominant height, α_{si} is the site-specific parameter for growth series i , whose starting value for fitting is the observed site index value at the specified base age, d_i is a dummy variable (being 1 for growth series i or otherwise 0), β is a vector for global parameters, t_j is any age, t_0 is any initial condition and e_{ij} is the error term. The sum of terms containing the dummy variables and site-specific parameter collapses for each growth series into a single parameter which is unique to each growth series. The dummy variable approach was selected in this study as it is considered appropriate for data sets of a moderate size.

Repeated measurements for the same individual lead to autocorrelation, possibly affecting the statistical significance of the parameters, which may be biased (Monserud, 1984). This autocorrelation is corrected by introducing an autoregressive procedure. In order to identify the autocorrelation structure, first we fitted the model and then plotted the residuals versus lag(p), where p represents the autocorrelation order. The structure chosen was the continuous autoregressive error structure (CAR(p)), which is considered to be appropriate for unevenly spaced longitudinal data (Gregoire et al., 1995; Zimmerman and Núñez-Antón, 2001; Diéguez-Aranda et al., 2005a). The general structure of the CAR(p) is as follows:

$$e_{ij} = \sum_{p=1}^n d_p \phi_p^k e_{ij-p} + \varepsilon_{ij} \quad (3)$$

where e_{ij} is the model error which follows an autoregressive procedure, d_p is 1 when $j > p$ and ϕ_p^k is the p continuous time autoregressive parameter where $k = t_{ij} - t_{ij-p}$ and $t_j > t_{j-p} \forall i$.

2.3. Functions tested

The relationship between the parameters and the observed site index was analysed through four of the most common base growth functions:

Hossfeld's polynomial function (1822),

$$Y = \frac{bt^c}{t^c + a} \quad (4)$$

the empirical model proposed by Schumacher (1939),

$$\ln Y = a + bt^c \quad (5)$$

and finally, two functions derived from growth theory were tested – the well known Richards' (1959) function,

$$Y = a(1 - \exp(-bt))^c \quad (6)$$

and the modification of Weibull's (1939) function proposed by Yang et al. (1978),

$$Y = a(1 - \exp(-bt^c)) \quad (7)$$

2.4. Model selection and evaluation

Evaluation should include model criticism and benchmarking (Vanclay, 1994). The first may be carried out observing the consistent biological behaviour of the model (Soares et al., 1995) and the second, through an independent data set (Snee, 1977). However, forest data are usually scarce and independent data not used in the fitting phase are normally lacking. Splitting methods divide the data into two subsets, but this is considered inappropriate if data are scarce (Burk, 1988) and results usually depend on partition methods. In situations of data scarcity, alternative procedures for model evaluation would be to either not benchmark the model until real independent data is available or utilize re-sampling methods, such as cross-validation or jack-knife techniques.

Re-sampling methods extract information of a given data type from within a sample of that data. In this study the model was fitted to all data except that for one tree (a set of observations) and the mean prediction error of the omitted growth series was then calculated. This procedure was repeated n times; n being the number of growth series in the sample. In this way, both the absolute and the relative values of the average prediction error are identified. The initial conditions for the evaluation statistic calculations were the average age and the corresponding height calculated using the algorithm proposed by Carmean (1972). The use of a particular point at which growth is measured as the initial condition series would produce a zero value for the error estimation at that point and mean error results would be lower than expected.

In fact, this procedure is similar to that used in PRESS statistics (Myers, 1990) except that a set of observations are dropped, i.e. the whole growth series, instead of one observation at a time. Barrio et al. (2006) referred to this validation technique as a double cross-validation approach.

The mean prediction error of the growth series was chosen as an indicator of model accuracy (Eqs. (8)–(9)).

Precision was evaluated through the mean absolute prediction error and root mean squared error of the growth series (Eqs. (10)–(13)). The model efficiency was calculated using the predicted value for all the observations (Eq. (14)).

$$e_i = \frac{\sum_{i=1}^k (H_i - \hat{H}_i)}{k} \quad (8)$$

$$e_{ev} = \frac{\sum_{i=1}^n e_i}{n} \quad (9)$$

$$abse_i = \frac{\sum_{i=1}^k |\hat{H}_i|}{k} \quad (10)$$

$$abse_{ev} = \frac{\sum_{i=1}^n abse_i}{n} \quad (11)$$

$$RMSEP_i = \sqrt{\frac{\sum_{i=1}^k (H_i - \hat{H}_i)^2}{k - p}} \quad (12)$$

$$RMSEP_{ev} = \frac{\sum_{i=1}^n RMSEP_i}{n} \quad (13)$$

$$MEF = 1 - \frac{\sum_{i=1}^s (H_i - \hat{H}_i)^2}{\sum_{i=1}^s (H_i - \bar{H}_i)^2} \quad (14)$$

where k is the number of observations within the same growth series, p is the number of parameters and n is the total number of growth series.

The evaluation was performed for all models. Graphical representations of fitted curves over data were finally performed to assess the applicability of the selected model.

2.5. Geographic variation

Comparison of regional growth patterns was made using two tests based on the likelihood-ratio method, the nonlinear extra sum of squares (Draper and Smith, 1981; Ratkowski, 1983) and the Lakkis–Jones test (in Khatree and Naik, 1995). The likelihood-ratio methods are the most appropriate in nested models where a set of parameters are shared by two models (Gregoire et al., 1995). The only requirement is to fit a full or complete model where a different set of parameters is needed for each of the regions considered, and a reduced model with a common set of parameters for all regions. The full model is obtained by expanding the parameters with $n - 1$ dummy variables, n being the number of regions involved.

The nonlinear sum of squares compares the statistics $F = [(SSE_r - SSE_f)/(df_r - df_f)]/[SSE_f/df_f]$, with an F -distribution for degrees of freedom $v = df_r - df_f$ and $u = df_f$ where df_r and df_f are reduced model and full model degrees of freedom, respectively. If $F > F_{crit}$ a model for each region is needed.

The Lakkis–Jones’ statistics is $L = (SSE_f/SSE_r)^{n/2}$, when homogeneity of parameters exists then $-2 \ln(L)$ follows a Chi-squared distribution with v degrees of freedom.

In order to evaluate the existence of geographic differences in growth patterns, the parameters which had previously been considered common to all regions were expanded with regional dummy variables in a full model.

The dynamic models were fitted using the PROC MODEL of SAS/ETS v 8 for Windows (SAS Inc., 1999).

3. Results

The relationship between site index at base age 70 years and individual model parameter estimates for each growth series are presented in Fig. 2. A visual inspection of the graphs revealed the potential relationships between the parameters and site productivity. A regression analysis identifies the significance of such relationships. The regression was of the type Parameters = $f(SI)$ and performed under three assumptions: linear,

inverse and quadratic variation. Table 2 shows the significance of each parameter at alpha 0.05.

The randomness in base age selection for the assessment of model parameters was defined by comparing the behaviour of parameters on the graph at different base ages (Fig. 3 shows an example for Hossfeld’s model).

The parameter corresponding to the asymptote in the Hossfeld’s model was found to vary with site productivity. A linear variation of type $b = b_1 + b_2X$ is considered when $b_2 = 1$ to avoid a zero solution, parameter a is also a varying parameter and is inversely proportional to site index, so the final model is equivalent to that presented by Cieszewski et al. (2006)

$$Y = \frac{(b + X)t^c}{t^c + a/X} \quad (15)$$

and the dynamic model is

$$H = H_0 \frac{t^c(t_0^c R_0 + e^b)}{t_0^c(t^c R_0 + e^b)} \quad (16)$$

where a , b and c are the parameters to be found

$$R_0 = H_0 - a + \left[(H_0 - a)^2 + 2H_0 \frac{e^b}{t_0^c} \right]^{0.5} \quad (17)$$

In the case of Schumacher’s model, parameter a varies linearly with the site index and b is inversely proportional to site index. Parameter a corresponds to the asymptote and it makes biologically sense that a linear relationship of the type $a = X$ exists to avoid a zero value in the parameter estimation (Cieszewski et al., 2006). Nevertheless, a relationship of the type $a = a_1 + a_2X$ would be equivalent to $a = X$ when a_1 and a_2 are allowed to be unequal to zero. Thus the growth equation is as follows:

$$\ln Y = X - \frac{b_1}{X} t^c \quad (18)$$

The equation to be solved is

$$X^2 - X \ln Y - b_1 t^c = 0 \quad (19)$$

Substituting for the initial conditions t_0 , H_0 :

$$X_0 = 0.5[\ln H_0 + ((\ln H_0)^2 - 4b_1 t_0^c)^{0.5}] \quad (20)$$

This expression is substituted in Eq. (19) to obtain the final dynamic model.

$$H = \exp\left(X_0 - \left(\frac{b_1}{X_0}\right) t^c\right) \quad (21)$$

The parameters of the base equation related to site index in Richards’ function according to the regression analysis are a and c in the linear and inverse assumption. However, solving both parameters at the same time would not produce a closed-form solution (Cieszewski, 2004). Parameter a corresponds to the asymptote and was substituted by the assumption $a = X$.

$$Y = X(1 - \exp(-bt))^c \quad (22)$$

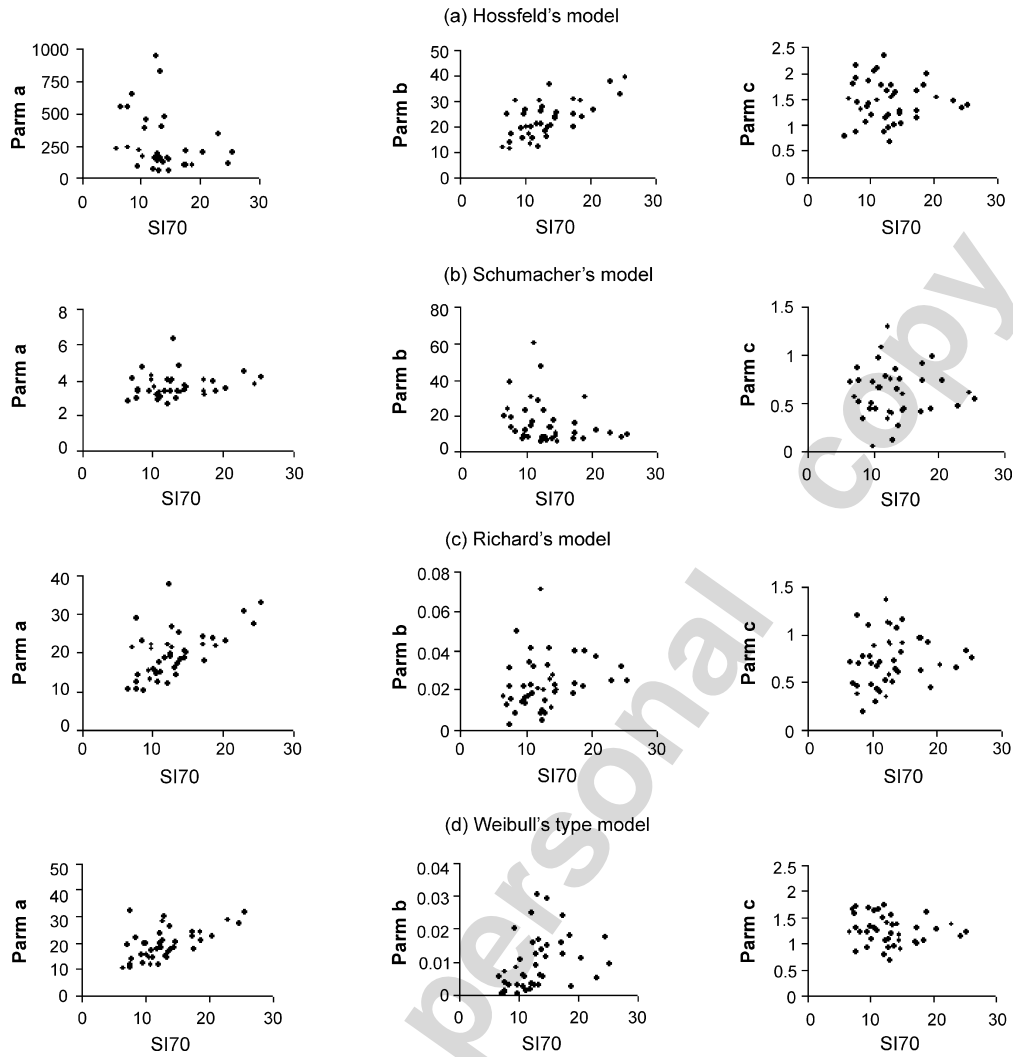


Fig. 2. Parameters trajectories along site index (BA = 70 years) using base equations.

Table 2
p-value of the regression analysis of parameters on actual site index at 70 years

Parameter	Model				
	Hossfeld's	Schumacher's	Richards'	Weibull's type	
Linear assumption					
a	0.0008	0.0111	0.0001	0.0001	
b	0.0001	0.0466	0.3103	0.0187 ^a	
c	0.2272	0.8599	0.0167	0.0144	
Inverse assumption					
a	0.0001	0.0051	0.0001	0.0001	
b	0.0001	0.0269 ^a	0.2323	0.0005	
c	0.0992	0.9447	0.0178	0.0095	
Quadratic assumption					
a	X	0.0022	0.2432	0.3394	0.5299 ^a
	X ²	0.0116	0.4195	0.7632	0.4147
b	X	0.6767 ^a	0.1691	0.4795	0.0072
	X ²	0.5189	0.2691	0.5876	0.0273
c	X	0.1239	0.5103	0.2080	0.0793
	X ²	0.1753	0.4904	0.3922	0.1734

^a The intercept is not significant.

When the model is solved, replacing the initial conditions, this is equivalent to an ADA formulation:

$$H = H_0 \left[\frac{(1 - \exp(-bt))}{(1 - \exp(-bt_0))} \right]^c \tag{23}$$

The Weibull type function has two varying parameters: *a*, corresponding to the asymptote and *b*, connected to the age of the inflection point. It can be seen from Fig. 2 that the most reasonable hypothesis supported by regression analysis is that both parameters are proportional to site quality in the following way:

$$\begin{aligned} a &= a_1 + a_2X \\ b &= b_1 + b_2X \end{aligned} \tag{24}$$

This model will be polymorphic with multiple asymptotes if $a_1 = 0$ and $a_2 = 1$ (Cieszewski, 2004). The final dynamic model takes the following form:

$$\ln H = X_0 + [b_1 + b_2X_0] \ln(1 - \exp(-t^c)) \tag{25}$$

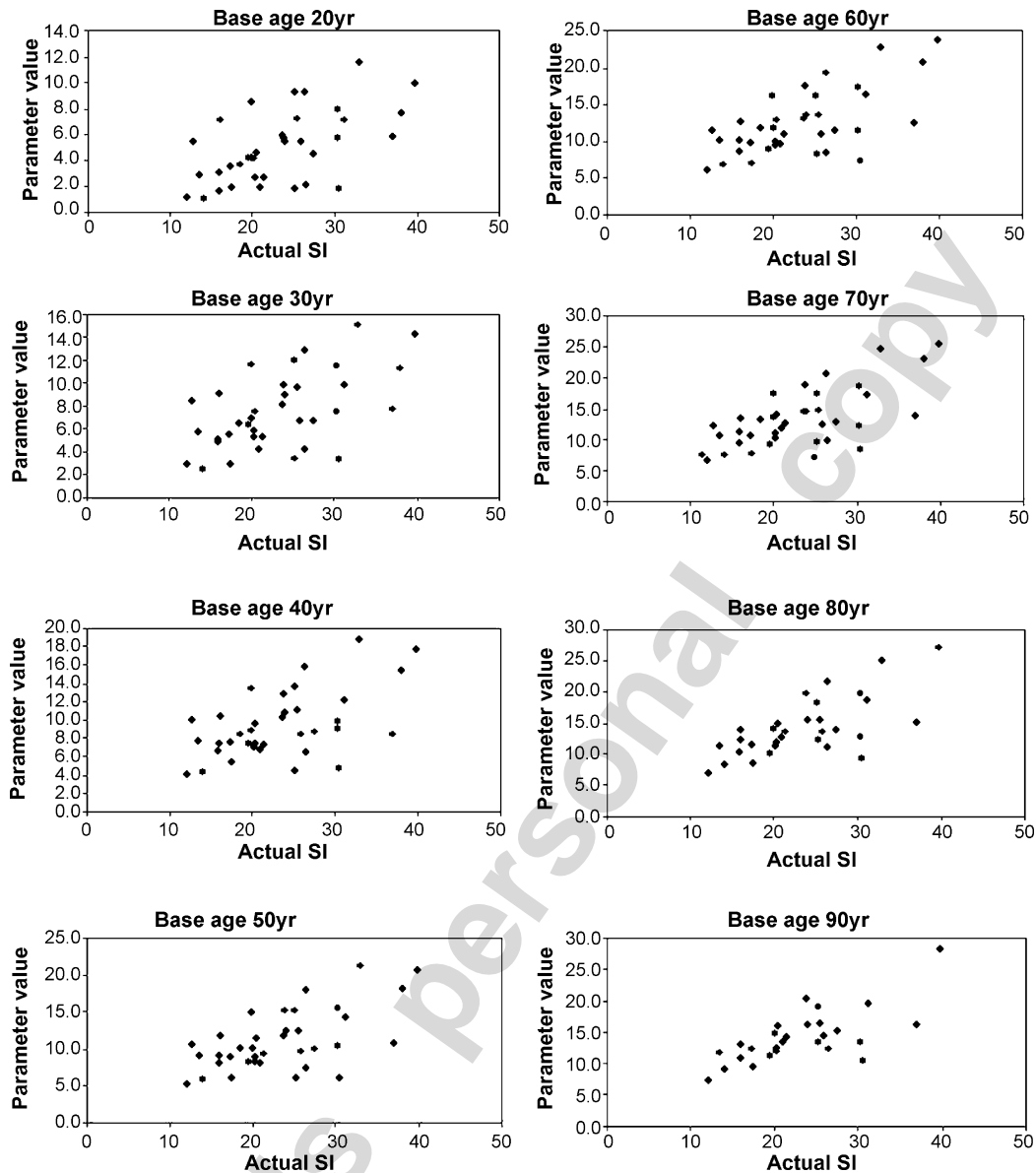


Fig. 3. Trajectory of the asymptote related parameter of Hossfeld's model versus actual site index using different base age.

where X_0 is

$$X_0 = \frac{\ln H_0 - b_1 \ln(1 - \exp(-t_0^c))}{1 + b_2 \ln(1 - \exp(-t_0^c))} \quad (26)$$

Once the dynamic models had been formulated and fitted, the analysis of residuals versus lag residuals showed that all models needed a second order autocorrelation (CAR(2)). Table 3 shows the parameters estimated and the fit statistics, including the value of the maximum asymptote or growth limit. Model selection was made on the basis of biological consistency, goodness of fit in terms of bias and precision, as well as on the graphical display of the data. All models performed adequately when comparing the MSE statistics, adjusted R^2 and bias in the fitting phase. The value of the asymptote estimated by each model was compared with the value for

the maximum total height reported by the Second National Forest Inventory (DGCN, 1998) for the species and region, which is 29 m in NPS, 35 m in CMS, 27 m in IMS and 28.8 m in SES, revealing that the value of the asymptote is extremely high in Schumacher's model when all regions are combined (77.91 m), especially in the IMS region (127.33 m) which takes it outside any practical application.

The evaluation reveals that the Weibull type model and Hossfeld's model perform adequately within an acceptable range (Table 4). The bias and accuracy of these two models are very similar. Again, biological soundness favours the selection of Hossfeld's model because the asymptotes, as well as the graphical representation of model versus data, are more realistic (Table 3).

The regional comparison indicated that different models are needed (Table 5). In the two by two comparisons, all regions

Table 3
Parameters and fitting statistics

Region	Model	Parameters					Asymptote (m) ^a	MSE	RMSE	Adj. R ²
		<i>a</i>	<i>b</i>	<i>c</i>	<i>b</i> ₁	<i>b</i> ₂				
All regions combined	Hossfeld's	14.45 (1.726)	8.418 (0.11516)	1.341 (0.028)			18.2–33.6	0.400	0.632	98.73
	Schumacher's			−0.447 (0.020)	34.550 (0.574)		27.9–77.9	0.401	0.633	98.72
	Richards'		−0.0183 (0.00097)	1.161 (0.030)			8.8–35	0.476	0.690	98.49
	Weibull's type				40.571 (4.031)	−7.637 (1.166)	14.9–41.7	0.401	0.623	98.87
NPS	Hossfeld's	21.54 (3.101)	6.42 (0.955)	1.271 (0.056)			22.1–25.1	0.398	0.631	98.41
	Schumacher's			−0.439 (0.038)	27.959 (0.805)		23.5–66.8	0.365	0.604	98.54
	Richards'		0.024 (0.0026)	1.128 (0.062)			11.4–22.7	0.472	0.687	98.36
	Weibull's type			0.275 (0.011)	76.94 (25.65)	−19.51 (7.82)	15.6–28.7	0.372	0.610	98.32
CMS	Hossfeld's	17.99 (3.36)	8.62 (0.29)	1.400 (0.05)			22.7–34.5	0.440	0.663	98.87
	Schumacher's			−0.471 (0.036)	37.859 (1.154)		27.8–77.5	0.460	0.679	98.93
	Richards'		0.022 (0.0018)	1.289 (0.057)			12.3–32.8	0.515	0.717	98.80
	Weibull's type			0.274 (0.009)	46.30 (9.07)	−8.76 (2.52)	20.5–42.5	0.452	0.672	98.95
IMS	Hossfeld's	14.46 (3.72)	8.60 (0.397)	1.313 (0.037)			19.2–32.9	0.275	0.525	98.63
	Schumacher's			−0.369 (1.263)	39.511 (1.263)		48.2–127.5	0.281	0.530	98.60
	Richards'		0.011 (0.0015)	1.112 (0.050)			18–41.9	0.307	0.554	98.48
	Weibull's type			0.241 (0.012)	38.72 (8.13)	−6.65 (2.364)	26.6–48.1	0.277	0.527	98.62
SES	Hossfeld's	17.63 (1.946)	7.86 (0.338)	1.421 (0.053)			20.1–27.6	0.364	0.595	98.84
	Schumacher's			−0.586 (0.0388)	38.165 (2.309)		17.94–53.19	0.337	0.581	98.90
	Richards'		0.0213 (0.00183)	1.235 (0.0667)			12.3–28.8	0.441	0.664	98.56
	Weibull's type			0.292 (0.008)	63.28 (13.20)	−14.77 (4.06)	17.6–31.3	0.334	0.579	98.91

^a The asymptote value is calculated with the lowest and the highest quality found in each region.

were found to be similar to IMS, which is considered to be path dispersion origin of the species after glaciations (Salvador et al., 2000). Precision varies according to the age class considered (Fig. 4). Both models are unbiased within the 20–40 years and over 80 years age classes, correcting the bias found in the previous model. Precision is higher when the regional model is used. However both models are still biased within the 40–60

years age class indicating that more specific features, apart from regionalization, contribute to growth pattern differentiation at this age.

Finally, the regional model selected consists of a dynamic form of Hossfeld's model where parameters *a* and *b* are considered to be site specific, according to the parameter assessment undertaken prior to dynamic formulation, whereas

Table 4
General model evaluation statistics

Region	Model	EF Jack-knife	Absolute prediction error	Mean prediction error	% mean error	% absolute error	RMSE
All combined	Hossfeld's	96.73	0.71	−0.15	−2.13	8.16	1.16
	Schumacher's	96.53	0.71	−0.15	−1.84	7.98	1.07
	Richards'	94.85	0.85	−0.29	−3.39	9.57	1.27
	Weibull type's	96.69	0.71	−0.14	−1.85	8.00	1.16
NPS	Hossfeld's	96.65	0.66	−0.25	−3.31	7.97	1.06
	Schumacher's	96.21	0.64	−0.24	−3.01	7.68	0.99
	Richards'	93.98	0.83	−0.38	−4.66	9.98	1.24
	Weibull's type	96.64	0.64	−0.24	−3.07	7.72	1.04
CMS	Hossfeld's	97.47	0.72	−0.14	−1.88	7.56	1.21
	Schumacher's	95.05	0.85	−0.15	−1.50	8.47	1.31
	Richards'	96.24	0.84	−0.23	−2.58	8.75	1.28
	Weibull's type	97.32	0.72	−0.13	−1.50	7.55	1.23
IMS	Hossfeld's	95.86	0.66	−0.13	−0.65	8.57	1.04
	Schumacher's	96.12	0.64	−0.17	−0.70	8.24	1.02
	Richards'	95.06	0.72	−0.28	−0.86	10.25	1.06
	Weibull's type	96.06	0.65	−0.14	−1.97	8.38	1.03
SES	Hossfeld's	97.48	0.63	0.05	−0.13	6.45	1.04
	Schumacher's	97.30	0.64	0.02	−0.11	6.48	0.94
	Richards'	95.51	0.80	−0.09	−1.37	8.04	1.20
	Weibull's type	97.30	0.63	0.05	−0.03	6.46	0.83

Table 5
Regional comparison using the Lakkis–Jones statistics (L) and nonlinear sum of squares (F)^a

Region compared	L	p -value	F	F_{crit}
All regions	27.06	<0.0001	8.08	2.61
NPS-CMS-IMS	34.18	<0.0001	15.36	3.01
NPS-CMS-SES	14.7	<0.0001	6.55	3.01
NPS-IMS-SES	11.05	<0.0001	4.93	3.01
CRS-IMS-SES	15.11	<0.0001	6.76	3.01
NPS-CMS	38.57	<0.0001	34.88	3.86
NPS-IMS	3.87	0.049	3.42	3.86
NPS-SES	9.97	<0.0001	8.89	3.86
CMS-IMS	1.85	0.1700	1.63	3.86
CMS-SES	11.33	0.0008	10.1	3.86
IMS-SES	2.96	0.0854	2.64	3.86

^a In bold: regions that may use one single equation.

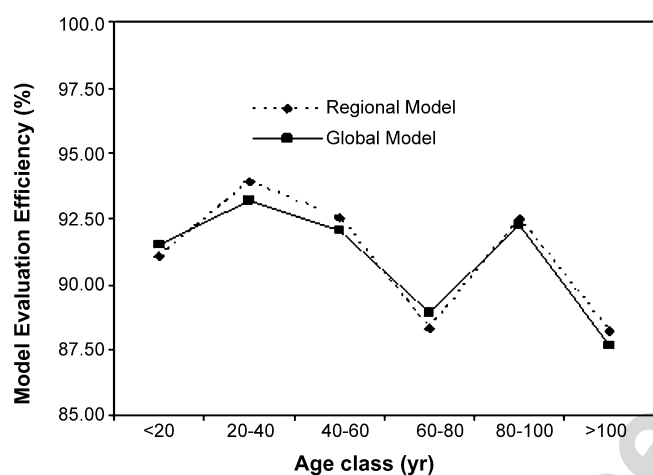


Fig. 4. Model efficiency for evaluation data for global and regional models across age classes.

parameter c is expanded on a regional basis, leading to a more flexible set of curves which are considered to be more appropriate for the species (Table 6). Fig. 5 shows the curves as well as the data used in the analysis.

4. Discussion

A well-behaved model should encompass true base-age invariance, polymorphism and asymptote, which, depending on the species, can vary or be common to all sites. The property of base-age invariance (BAI) is probably the most controversial of all the assumptions of growth modelling, as stated by Bailey

and Cieszewski (2000), who discussed the use of BAI in several articles and found that many did not possess true base-age invariance. Cieszewski et al. (1998) identified two conditions required to achieve base age invariant models: (1) the model specification must be algebraic and (2) the model coefficients must be estimated regardless of the choice of base age. In this study, the first condition is complied with by using the generalization of the algebraic difference equations proposed by Cieszewski and Bailey (2000), and the second one by using a dummy variable approach in the fitting phase (Cieszewski, 2004).

The suggestion behind the Generalized Algebraic Difference Approach (GADA) is that there is more than one site-specific parameter. These site-parameters might be considered unobservable variables or functions of a variable (X) which encompasses all the factors affecting tree growth, and is termed the 'growth intensity factor' (GIF) (Cieszewski and Bailey, 2000). Therefore, the data, or rather, the actual information about what is going on in the forest, can be seen as the expression of many factors: climate, soil, landform, genetics or competition. Models account for such multifactor expression through parameterization.

Every parameter in a dominant height growth model must have its own biological significance. There are parameters related to the asymptote (maximum growth potential) and others related to shape. Studies concerning parameter behaviour have been carried out in forest research to determine the general structure of growth models (Zeide, 1993) as well as to establish and solve parameter requirements (Shvets and Zeide, 1996) and to identify the best method for parameter estimation based on individual trends in the data (Cieszewski, 2000).

Three assumptions regarding the relationship between base equation parameters and actual site index were tested using regression analysis and graphical display. Carvalho and Parresol (2005) studied the relationship of the parameters in the Richards' function using ANOVA according to site index classes and found that only parameter a (corresponding to the asymptote) was correlated with site index. They substituted this parameter for the X variable and developed a model with ADA formulation. However, according to Zeide (2004) a regression analysis of parameters in observed site index may account for local variability and should be applied if the aim is to determine the type of relationship which exists between site index and the parameters. Variation in more than one parameter was detected in all models for Mediterranean

Table 6
Regional model parameters and main statistics for the fitting and evaluation phase

Model parameters						Fitting			Evaluation					
a	b	c	c_1	c_2	c_3	MSE	RMSE	Adj. R^2	Evaluation EF	Absolute prediction error	Mean prediction error	% mean error	% absolute error	RMSE
15.34	8.49	1.528	-0.475	-0.169	-0.233	0.39	0.626	98.75	96.8	0.71	-0.18	-2.53	8.14	1.07
(1.34)	(0.12)	(0.052)	(0.066)	(0.066)	(0.086)									

$c_1 = 1$, $c_2 = 0$ and $c_3 = 0$ when region is NPS. $c_1 = 0$, $c_2 = 1$ and $c_3 = 0$ when region is CMS. $c_1 = 0$, $c_2 = 0$ and $c_3 = 1$ when region is IMS. $c_i = 0$ when region is SES.

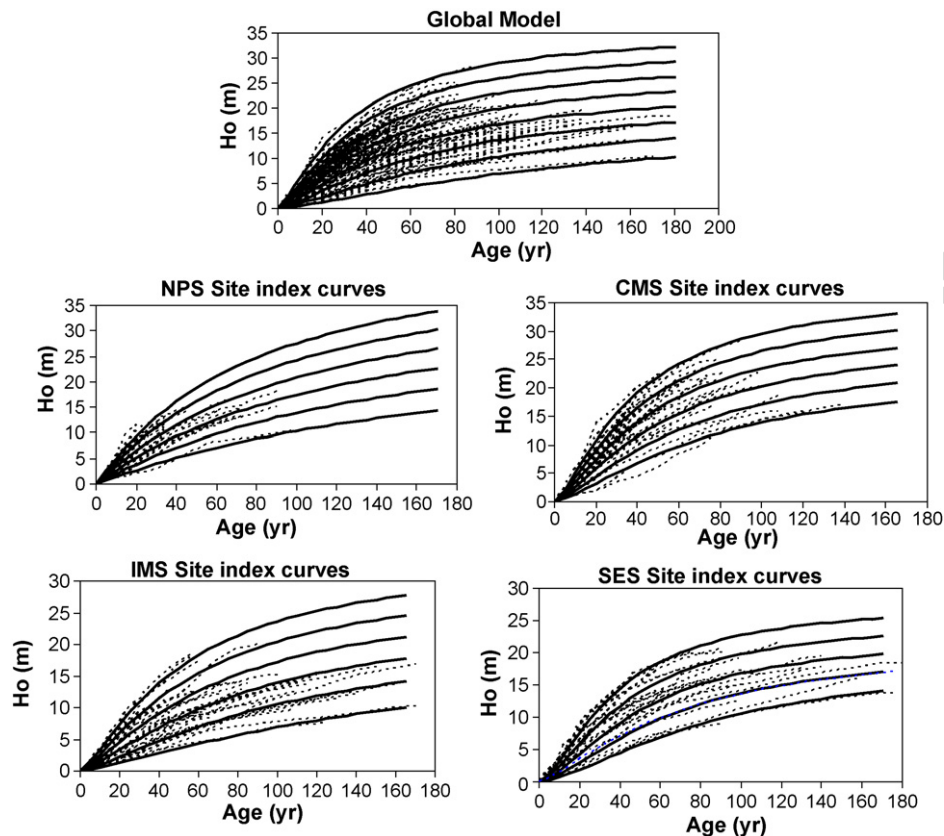


Fig. 5. Global and regional dynamic dominant height-site index model for MMP in Iberian Peninsula.

Maritime Pine stands in the Iberian Peninsula. The asymptote parameters were found to be linearly dependent on site index leading to varying asymptote, whereas those parameters related to growth rate or point of inflection were inversely proportional, indicating that faster growth rates occur earlier in good stands than in poor ones. This reflects polymorphism in the growth pattern of the species.

According to Goelz and Burk (1996), the height at a given reference age is an estimate of true site index since it is considered the result of the intrinsic growth-pattern of the site. Parameter estimates should reflect this factor. In fact, true site index-parameter response will always be similar regardless of the base age considered (Fig. 3), which supports the assumption that dominant trees selected according to Assmann's criterion (Assmann, 1971) would have been dominant over their whole lifespan.

It was found that parameter assessment prior to GADA formulation helps in the dynamic formulation. However, the fact that the same kind of site index-parameter hypothesis was tested for the whole distribution of the species may have masked the real parameter variation within each region. A deeper regional insight would only be possible with a large database, which unfortunately is not available. However, model efficiency is sufficiently high for the proposed model to be trusted (Table 6 and Fig. 4).

The assessment of parameter significance has become a prerequisite of model building using generalized difference

formulation since dynamic growth models have both specific as well as global parameters. The identification of specific parameters must assure a closed-form solution. This implies that any parameter dependency is unacceptable. Sometimes, parameters considered to be global may also display a weak relationship to site conditions (or not so weak, such as in the Weibull type model), but the solvability of the equation obliges us to consider them common to all sites. This might be considered a weakness in the generalized algebraic difference approach since we are forsaking certain information concerning growth patterns in order to respect the mathematical aspect of model formulation.

However, the expansion of global parameters according to regional factors does not violate any mathematical rule and gives more flexibility to model behaviour by taking into account geographic variation. The parameter assessment was conducted using all the data, attempting to identify differences in the parameter values according to differences in site productivity. If these differences exist, the parameter is considered to be site specific, otherwise it is deemed to be global. However, the term 'global' does not necessarily mean that it must be common to all sites in every aspect of the growth process. In our study we found that parameter c in Hossfeld's function is not related to site productivity, but its expansion according to regional factors indicated that a sort of site dependency exists which might be interpreted in terms of interregional variability. If this dependency appears to

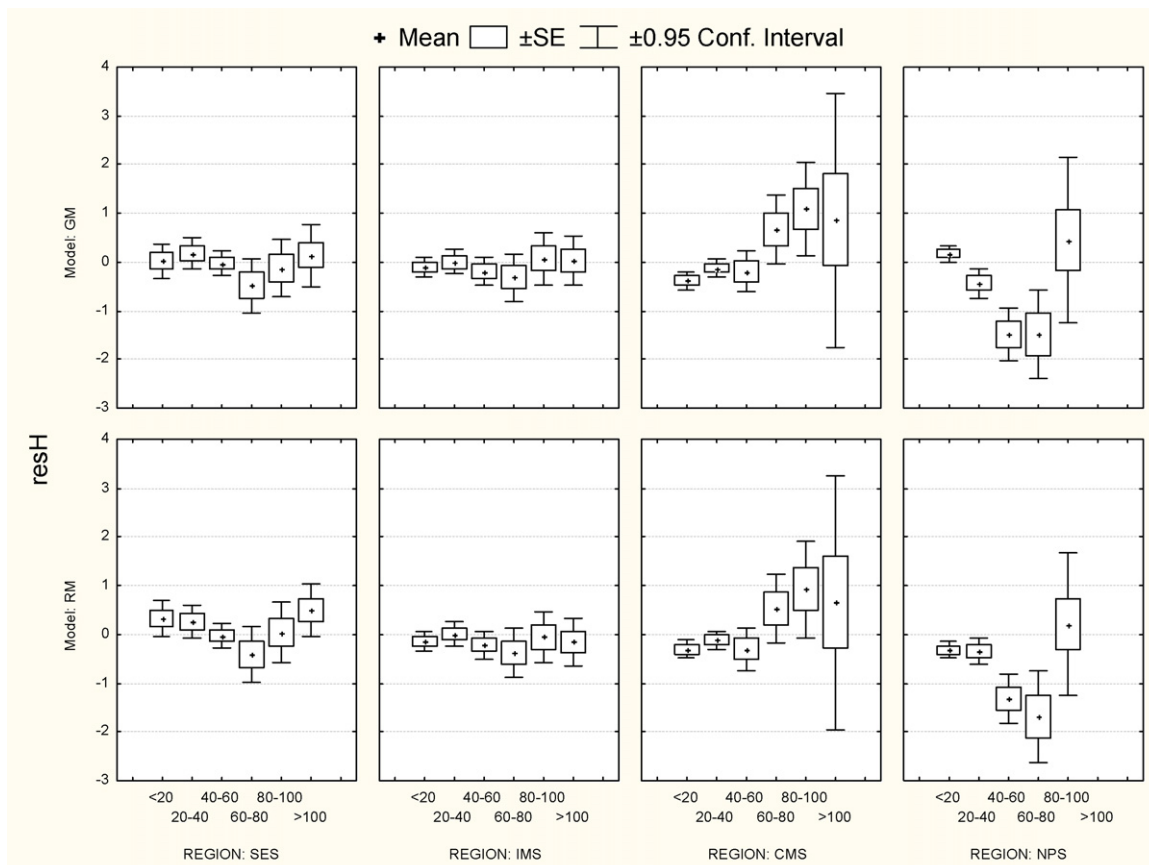


Fig. 6. Evaluation bias performance through age classes and regions for global model (GM) and regional model (RM).

be related to soil attributes, climate or genetics should be taken into consideration since growth is associated with environmental and genetic aspects, as well as their interaction.

The differences found in growth patterns from one region to another indicate that stands located at large distances from the hypothetical origin of dispersion for the species (NPS, CMS, SES) follow different growth trajectories, possibly attributable to isolation, genetic differentiation and climatic variability. However, all these regions display a somehow similar growth pattern to that found at the origin of dispersion, near IMS (Salvador et al., 2000; González-Martínez et al., 2001). The similarity between the NPS, CMS and SES regions facilitates an accurate global model. In fact, both the global and the regional models are sufficiently accurate to be applied (depending on forest manager's requirements). If the intention is to compare the productivity of different regions, then the global model will be accurate enough. However, if the management plan focuses on a specific region, it would be better to use the regional model.

Any particularity in growth conditions within one specific region may augment the differences among various regions and could affect the performance of a general model. This is the case of the Northern Plateau Stands where provenances are commonly poorly-formed and the straightness of stems is not assured, resulting in problematic tree height measurement. This is one possible cause of the bias found in both models. However, another plausible biological hypothesis is that dominant tree growth tends to slow down around this age, often because of environmental conditions, especially those

related to soil properties. In the NPS region, it is easy to find abiotic soil horizons resulting from the compacting associated with clay and silt, which are sometimes very superficial and limit the available soil for roots (Nicolás and Gandullo, 1967). These conditions change the normal growth trajectory which the trees would otherwise follow under appropriate soil conditions. Furthermore, sandy soils with deep soil profiles are also common in NPS, which increases the variability of the growth pattern within this region. In fact, when both models are applied to all the regions, the bias in the performance evaluation is only evident in the northern plateau stands (Fig. 6).

The comparison of the models demonstrated that there is a requirement for separated models. The real causes of the differentiation are beyond the scope of an empirical study. Models that describe interaction with other variables, such as climate and soil properties, would provide valuable information for managers concerned about local variability.

5. Conclusions

The analysis of parameter-site index relationships has facilitated the identification of an appropriate GADA expression by reducing the number of alternatives and assisting growth hypothesis contrasts. It is possible to evaluate regional differences with the GADA formulation by expanding global parameters that could explain geographic variation. In this sense, global parameters become interregional parameters as opposed to common parameters defined in an earlier phase,

especially when global parameters are defined in such a way that solvability is assured.

Acknowledgements

Authors want to express their deep gratitude to those who have contributed to the installation, measurement and maintenance of permanent sample plot network. Special thanks to Dr. Ulises Diéguez for his comments on GADA and dummy variable approach. A. Bravo-Oviedo wants to thank Dr. Margarida Tomé who helped in the final process of the model development, lead at her laboratory in Portugal. This work was partly funded by research projects OT 03-002 and AGL 2004-007094-C02-02.

References

- Alía, R., Moro, J., Denis, J.B., 1997. Performance of *Pinus pinaster* provenances in Spain: interpretation of the genotype by environment interaction. *Can. J. For. Res.* 27, 1548–1559.
- Alía, R., Martín, R., de Miguel, J., Galera, R.M., Agúndez, D., Gordo, J., Salvador, L., Catalán, G., Gil, L.A., 1996. Regiones de Procedencia. *Pinus pinaster* Ait. DGCN, Madrid.
- Assmann, E., 1971. *The Principles of Forest Yield Study*. Pergamon Press.
- Bailey, R.L., Cieszewski, C.J., 2000. Development of a well-behaved site-index equation: jack pine in north-central Ontario: comment. *Can. J. For. Res.* 30, 1667–1668.
- Bailey, R.L., Clutter, J.L., 1974. Base-Age invariant polymorphic site curves. *For. Sci.* 20, 155–159.
- Barrio, M., Castedo, F., Diéguez-Aranda, U., Álvarez, J.G., Parresol, B.R., Rodríguez, R., 2006. Development of a basal area growth system for maritime pine in northwestern Spain using the generalized algebraic difference approach. *Can. J. For. Res.* 36, 1461–1474.
- Bravo-Oviedo, A., Rfo, M., Montero, G., 2004. Site index curves and growth model for Mediterranean maritime pine (*Pinus pinaster* Ait.) in Spain. *For. Eco. Manage.* 201, 187–197.
- Burk, T.E., 1988. Prediction error evaluation: preliminary results. In: Wensel, L.C., Biging, G.S. (Eds.), *IUFRO Conference on Forest Simulation Systems*. University of California, Berkeley, p. 420.
- Carmean, W.H., 1972. Site index curves for upland oaks in the Central States. *For. Sci.* 2, 109–121.
- Carvalho, J.P., Parresol, B.R., 2005. A site model for Pyrenean oak (*Quercus pyrenaica*) stands using a dynamic algebraic difference equation. *Can. J. For. Res.* 35, 93–99.
- Cieszewski, C.J., 2000. Practical methods for estimating non-biased parameters in self-referencing growth and yield models. In: Daniel, B. (Ed.), *Warnell School of Forest Resources*. University of Georgia, Athens, p. 11.
- Cieszewski, C.J., 2004. GADA derivation of dynamic site equations with polymorphism and variable asymptotes from Richards. In: Weibull and other Exponential Functions, *Plantation Management Research Cooperative*, Athens, p. 10.
- Cieszewski, C.J., Bailey, R.L., 2000. Generalized Algebraic Difference approach: theory based derivation of dynamic site equations with polymorphism and variable asymptotes. *For. Sci.* 46, 116–126.
- Cieszewski, C.J., Bailey, R.L., Borders, B.E., Brister, G.H., Shiver, B.D., 1998. Base-age invariance and inventory projections. In: *Integrated Tools Proceedings*, Boise, Idaho, pp. 481–493.
- Cieszewski, C.J., Zasada, M., Strub, M., 2006. Analysis of different base models and methods of site model derivation for Scots pine. *For. Sci.* 52, 187–197.
- Danjon, F., 1994. Stand features and height growth in a 36-year-old maritime pine (*Pinus pinaster* Ait.) provenance test. *Silvae Genetica* 43, 52–62.
- DGCN, 1998. *El Segundo Inventario Forestal Nacional*. MMA-DGCN, España, Madrid.
- Diéguez-Aranda, U., Burkhart, H.E., Rodríguez-Soalleiro, R.L., 2005a. Modeling dominant height growth of radiata pine (*Pinus radiata* D. Don) plantations in north-western Spain. *For. Eco. Manage.* 215, 271–284.
- Diéguez-Aranda, U., Álvarez-González, J.G., Barrio Anta, M., Rojo Alboreca, A., 2005b. Site quality equations for *Pinus sylvestris* L. plantations in Galicia (northwestern Spain). *Ann. For. Sci.* 62, 142–152.
- Draper, N.R., Smith, H., 1981. *Applied Regression Analysis*. John Wiley & Sons, New York.
- Elena-Rosselló, R., Castejón, M., Sánchez, F., Tella, G., Sánchez-Palomares, O., González-Adrados, J.R., Allué-Andrade, J.L., 1997. *Clasificación Biogeoclimática de España peninsular y Balear – Atlas Clateres*. Secretaría General Técnica, INIA.
- García, O., 2005. Comparing and combining stem analysis and permanent sample plot data in site index models. *For. Sci.* 51, 277–283.
- Goelz, J.C.G., Burk, T.E., 1996. Measurement error causes bias in site index equations. *Can. J. For. Res.* 26, 1585–1593.
- González-Martínez, S.C., Slavador, L., Agúndez, D., Alía, R., Gil, L., 2001. Geographical variation of gene diversity of *Pinus pinaster* Ait. in the Iberian Peninsula. In: Müller-Starck, G.a.S, R. (Eds.), *Genetic Response of Forest Systems to Changing Environmental Conditions*. Kluwer Academic Publishers, Dordrecht, Boston, London, pp. 161–171.
- Gregoire, T.G., Schabenberger, O., Barret, J.P., 1995. Linear modelling of irregularly spaced, unbalanced, longitudinal data from permanent-plot measurements. *Can. J. For. Res.* 25, 137–156.
- Hossfeld, J.W., 1822. *Mathematik für Forstmänner. Ökonomen und Cameralisten*, Gotha, p. 4.
- Hummel, F.C., Locke, G.M., Jeffers, J.N., Christie, J.M., 1959. Code of sample plot procedure. In: *Forestry Commission*.
- Khatree, R., Naik, D.N., 1995. *Applied Multivariate Statistics with SAS Software*. SAS Institute Inc., Cary, NC.
- Monserud, R.A., 1984. Height growth and site index curves for inland Douglas-fir based on stem analysis data and forest habitat type. *For. Sci.* 30, 943–965.
- Montero, G., Madrigal, G., Ruiz-Peinado, R., Bachiller, A., 2004. Red de parcelas 20 experimentales permanentes del CIFOR-INIA. In: *For. S.E.C. (Ed.), Reunión de 21 Modelización Forestal*. Soc. Esp. Cien. For., Palencia, pp. 229–236.
- Morgenstern, E.K., 1996. *Geographic Variation in Forest Trees. Genetics Basis and Application of knowledge in Silviculture*. UBC Press.
- Myers, R.H., 1990. *Classical and Modern Regression with Applications*. Wadsworth Publishing Company, p. 488.
- Nicolás, A., Gandullo, J.M., 1967. *Ecología de los pinares Españoles I. Pinus pinaster* Ait. Ministerio de Agricultura. Dirección General de Montes. In: *Caza y Pesca Fluvial, I.F.I.E.*, Madrid.
- Pita, P.A., 1967. *Clasificación provisional de las calidades de la estación en las masas de P. pinaster Sol. y P. uncinata Ram. de la península Ibérica*. *Anales del i.F.I.E.*, pp. 125–138.
- Ratkowski, D.A., 1983. *Nonlinear Regression Modeling. A Unified Practical Approach*. Marcel Dekker, Inc., New York, Basel.
- Ribeiro, M.M., Plomion, C., Petit, R., Vendramin, G.G., Szmidi, A.E., 2001. Variation in chloroplast single-sequence repeats in Portuguese maritime pine (*Pinus pinaster* Ait.). *Theor. Appl. Genet.* 102, 97–103.
- Richards, F.J., 1959. A flexible growth function for empirical use. *J. Exp. Bor.* 10, 290–300.
- Salvador, L., Alía, R., Agúndez, D., Gil, L., 2000. Genetic variation and migration pathways of maritime pine (*Pinus pinaster* Ait.) in the Iberian Peninsula. *Theor. Appl. Genet.* 100, 89–95.
- Schumacher, F.X., 1939. A new growth curve and its application to timber yield studies. *J. For.* 37, 819–820.
- Shvets, V., Zeide, B., 1996. Investigating parameters of growth equations. *Can. J. For. Res.* 26, 1980–1990.
- Snee, R.D., 1977. Validation of regression models: methods and examples. *Technometrics* 19, 415–428.
- Soares, P., Tomé, M., Skovsgaard, J.P., Vanclay, J.K., 1995. Evaluating a growth model for forest management using continuous forest inventory data. *For. Ecol. Manage.* 71, 251–265.
- Tait, D.E., Cieszewski, C.J., Bella, I.E., 1988. The stand dynamics of lodgepole pine. *Can. J. For. Res.* 18, 1255–1260.

- Vanclay, J.K., 1994. Modelling Forest Growth and Yield. CAB International, Wallingford.
- Vendramin, G.G., Anzidei, M., Madaghiele, A., Bucci, G., 1998. Distribution of genetic diversity in *Pinus pinaster* Ait. as revealed by chloroplast microsatellites. *Theor. Appl. Genet.* 97, 456–463.
- Weibull, W., 1939. A statistical theory of the strenght of material. In: Ingenioersveten-skapskad.
- Yang, R.C., Kozak, A., Smith, J.H.G., 1978. The potential of Weibull-type functions as flexible growth curves. *Can. J. For. Res.* 8, 424–431.
- Zeide, B., 1993. Analysis of growth equations. *For. Sci.* 39 (3), 594–616.
- Zeide, B., 2004. Intrinsic units in growth modeling. *Ecol. Model.* 175, 249–259.
- Zimmerman, D.L., Núñez-Antón, V., 2001. Parametric modelling of growth curve data: an overview. *Test* 10, 1–73 (with discussion).

Author's personal copy