

Dynamic growth and yield model for Black pine stands in Spain

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Abstract

In a forestry context, modelling stand development over time relies on estimates of different stand characteristics obtained from equations which usually constitute a multivariate system. In this study we have developed a stand growth model for even-aged stands of Black pine (*Pinus nigra* Arn.) in Spain. The 53 plots used to fit the equations came from the permanent sample plot network established by the Forest Research Centre (INIA) in 1963 and 1964 in the main distribution regions of Black pine. The model is made up of a system of equations to predict growth and yield in volume and basal area. In the fitting phase we took into account the correlation between the measurements within the same plot and the cross-equation residual correlations. The model incorporates a control function to estimate the thinning effect and a function for predicting the reduction in tree number due to regular mortality. In addition, we use the three parameter Weibull distribution function to estimate the number of trees in each diameter class by recovering the parameters using the moment method. The developed model is useful for simulating the evolution of even-aged stands with and without thinnings and allows the estimation of number of trees by diameter classes.

Key words: stand growth model; system of equations; seemingly unrelated regression; *Pinus nigra*; Weibull distribution function.

Resumen

Modelo dinámico de crecimiento y producción para rodales de *Pinus nigra* Arn. en España

En la modelización forestal, el crecimiento de un rodal se expresa por medio de la evolución en el tiempo de diferentes características dasométricas estimadas a partir de ecuaciones que habitualmente constituyen un sistema de datos multivariante. En el presente trabajo se ha elaborado un modelo de crecimiento para rodales regulares de pino laricio (*Pinus nigra* Arn.) en España. Las 53 parcelas utilizadas para ajustar las ecuaciones pertenecen a la red de parcelas permanentes establecidas por el Instituto Nacional de Investigación y Tecnología Agraria y Alimentaria (INIA) en los años 1963 y 1964 en las principales zonas de distribución de pino laricio. El modelo se compone de un sistema de ecuaciones para predecir el crecimiento y la producción del volumen y el área basimétrica. En su ajuste se tuvo en cuenta la correlación entre los datos dentro de una misma parcela y la correlación de los residuos entre las distintas ecuaciones del sistema. El modelo incorpora una función de control para estimar el efecto de las claras y una función para predecir la reducción del número de árboles debido a la mortalidad regular. Además, utilizamos la función de distribución de Weibull con tres parámetros para estimar el número de árboles en cada clase diamétrica recuperando los parámetros por el método de los momentos. El modelo desarrollado es útil para simular la evolución de rodales regulares con y sin intervención de claras y permite la estimación del número de árboles por clases diamétricas.

Palabras clave: modelo de crecimiento de rodal; sistema de ecuaciones; *seemingly unrelated regression*; *Pinus nigra*; función de distribución de Weibull.

Introduction

Yield tables can be considered the oldest and most robust approach for predicting growth and yield of forest stands, but they are limited by their static character

which implies the need to be applied to stands with a silvicultural regime similar to that of the yield table. Due to having growth data, the improvements in statistical software and the need for more detailed information, yield tables are being replaced by dynamic

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growth models, more flexible because they can simulate different management alternatives under different densities. If we focus on pine species of Mediterranean forest in which we may classify *P. nigra* Arn. in Spain, this has happened for *Pinus halepensis* Mill., *Pinus pinea* L. and *Pinus sylvestris* L. (Bravo *et al.*, 2011). However, until recently the unique growth models for *P. nigra* in Spain were yield tables (Gómez Loranca, 1996; Bautista *et al.*, 2001). Palahí and Grau (2003), to provide an alternative to the yield tables that had been traditionally used, tested the applicability of a preliminary tree-level distance-independent growth model made from six plots of the INIA network of permanent sample plots.

It is considered that, at least for even-aged and mono-specific stands, stand models using information usually collected on forest inventories are a suitable alternative compared to individual tree models (Vanclay, 1994; García, 2003). The stand development can be properly estimated from the adequate characterization of its current state by a set of stand-level variables (e.g., dominant height, basal area, number of trees) at that time (García, 1994). Assuming that these state variables summarize the historical events that could affect the future stand development, the future state is estimated from the initial situation updating the variables by means of equations in differential or integral form (García, 1988).

Taking this into account, the aim of this work was to develop a stand growth model with a dynamic approach for *P. nigra* in Spain. The model consists of a system of equations to predict volume and basal area, a function to estimate the quadratic mean diameter after a thinning and a function to estimate the reduction in the number of trees due to regular mortality. The model uses the Weibull probability function to model diameter distributions and a disaggregation system to estimate the Weibull's parameters as functions of stand characteristics.

Material and methods

The modelling data were obtained from 53 plots of the permanent sample plot network established by the Forest Research Centre (INIA-CIFOR) in even-aged *P. nigra* stands in 1963 and 1964. Plots are localized throughout three of the main distribution regions of *P. nigra* in Spain: Cazorla mountain range (22 plots), Serranía de Cuenca (22 plots) and the mountains that surround the southern margin of the Ebro valley

(9 plots). There are a total of 324 inventories and the number of inventories per plot ranges from 2 to 8. The missing tree heights in each plot were predicted using height-diameter models. The candidate height equations included the models of Curtis (1967) which use tree diameter and stand age as independent variables. Total volume of trees were calculated using the model by Martínez-Millán *et al.* (1993) for *P. nigra*. In most of plots light and moderate thinnings from below have been carried out ($G_{removed}/G_{before} = 10\% \pm 9\%$). For the estimation of site index we used the model of *P. nigra* developed by Martín-Benito *et al.* (2008). Summary statistics including the mean, minimum, maximum, and standard deviation of the main stand variables for total plots are shown in Table 1.

Basal area and volume

Initially were considered for evaluation in this study three compatible systems of equations which have already been published (Clutter, 1963; Sullivan and Clutter, 1972; Piennaar and Harrisson, 1989). The system which showed better results and has been used in this study was Sullivan and Clutter's system. This system allows us to estimate future volume and basal area as a function of site index, initial basal area and the initial and projected ages.

Because data used for modelling are taken repeatedly through time on trees in the same plots, within plot errors are expected to be serially correlated. To

Table 1. Summarised plot characteristics used for model development

Variable	Minimum	Average	Maximum	s.d.
<i>t</i>	31.0	84.9	175.0	34
<i>SI</i>	8.4	18.0	26.8	4.6
<i>Ho</i>	8.4	17.7	30.5	4.6
<i>N</i>	200	1,189	4,308	731
<i>dg</i>	10.5	24.4	52.2	7.5
\bar{d}	10.0	23.5	50.8	7.3
d_{min}	5.1	10.4	26.8	4.3
<i>G</i>	18.4	44.1	86.6	12.1
<i>V</i>	68.1	366.0	983.9	173.9

t: age (years); *SI*: site index (m, dominant height at age 80 year calculated according Martín-Benito *et al.* (2008) model); *G*: basal area (m²/ha); *N*: stem number per hectare; *dg*: mean quadratic diameter (cm); *Ho*: dominant height (m); d_{min} : minimum diameter (cm); \bar{d} : average diameter (cm); *V*: volume (m³/ha); s.d.: standard deviation.

correct this autocorrelation we have modelled the error term using a first order autoregressive error structure (Gregoire *et al.*, 1995).

This system of equations is referred as a seemingly unrelated regression system since only one dependent variable occurs in each equation. After finding that residuals between different equations were correlated, this cross-equation error correlations were included in a generalized least squares procedure using SUR (Zellner, 1962). The three equations constituting the system with the restriction on the parameters to ensure that the system is compatible (Sullivan and Clutter, 1972) and including an autoregressive error structure in each equation were fitted simultaneously using PROC MODEL and the SUR option in SAS/ETS® (SAS Institute, 2008).

Thinning

Thinnings usually happen at discrete points in time, causing an instantaneous change of state. Therefore, we can model the stand development between thinnings as a system where the exchange rate is just a function of their current situation (García, 1988) and a separate function may represent changes in state variables caused by thinnings (Río and Montero, 2001; Bravo-Oviedo *et al.*, 2004). They can be specified in terms of removed or remaining trees per hectare or they can be expressed in terms of stand basal area, usually as a percentage, to estimate then the number of trees. When both removed trees and basal area are known stand variables after thinning can be calculated directly. When only one of the two variables is known we adjusted an equation to estimate quadratic mean diameter after thinning from the quadratic mean diameter before thinning and the thinning intensity (Álvarez-González, 1997). To do this we have used the recorded data from inventories where thinning were applied. Parameters were estimated in PROC REG in SAS/STAT® (SAS Institute, 2009).

Mortality

To determine the evolution of stands between two successive inventories where there has been no thinning we have fitted a mortality equation. Zhao *et al.* (2007) set out various mortality functions expressed in algebraic differences. Using inventories where thinnings had not been carried out we fitted ten of these func-

tions. Model parameters estimates were obtained using PROC NLIN in SAS/STAT® (SAS Institute, 2009).

Diameter distribution

Here, we use the three parameter Weibull distribution and the moment method to obtain the diameter distribution from diameters sampled and to recover the parameters from stand variables. The routine solves a moment-based three parameter Weibull system of equations where the location parameter a was set equal to $0.5 d_{min}$ (Knoebel *et al.*, 1986). The stand variables used in the Weibull parameter recovery method are moments of the diameter distribution. The first moment is estimated by the arithmetic mean diameter of the stand, and the second moment by the quadratic mean diameter. In this study, the diameters were grouped in diameter classes by 5 cm increments.

The estimation of the stand variables required to fit the diameter distribution when there is no diameter inventory was based on a system of two equations to estimate the mean and minimum diameter from stand variables obtained from the growth models described above. Quadratic mean diameter can be obtained from the basal area equation. Restrictions on equations fitted were imposed because minimum diameter is lower than mean diameter and mean diameter of a stand is always lower than or equal to the quadratic mean diameter (Mateus and Tomé, 2011). Initially these two equations were fitted individually to select the regressor variables in each equation. The d_{min} equation showed heteroscedasticity problems. To deal with this a weighting factor was used. The error variances were modelled as a power function of d , $\sigma^2 \propto d^q$ using the method proposed by Harvey (1976) to obtain an estimate of q . Because mean diameter appears both as dependent and independent in the functions, three stage least square method (Zellner and Theil, 1962) was used to simultaneously estimate regression parameters of equations to predict minimum and mean diameter including the weighting factor as in the individual fitting. We used PROC MODEL in SAS/ETS® (SAS Institute, 2008).

Equation evaluations

Before calculating the statistics to evaluate the system of equations the variables $\ln(V_1)$, $\ln(G_2)$ and $\ln(V_2)$ predicted were transformed into metric scale. This

transformation from logarithmic to metric scale is not direct but requires a correction of the bias (Baskerville, 1972). To do this we used the ratio proposed by Snowdon (1991) between the arithmetic mean of observed values of the variable in question and the arithmetic mean of values in metric scale estimated by the model.

Quantitative evaluation involves the characterization of the model error and calculating the adjusted coefficient of determination (R_{adj}^2). Plots of observed against predicted values and predicted against residuals were checked. To detect possible trends in the residuals, they were represented by age class. PRESS residuals were obtained to compute mean of PRESS residuals and mean of absolute PRESS residuals. In the diameter distribution two goodness of fit statistics were computed for each plot-age combination: the Kolmogorov-Smirnov statistic and Reynolds *et al.* (1988) error index (EI).

Results and discussion

Basal area and volume

Residual plots showed no clear pattern indicating that the variance is homogeneously distributed and that the model predictions are not biased. All equations (Table 2, eqs. 1, 2 and 3) explained high percentage of the total variability of the data. The age range of our database ranges from 31 years to 175 years so forecasts outside this age range should be made with caution, at least until these equations are not validated and/or refitted with new data covering these new situations. It should be noted the presence of underrepresented situations. Thus, we find few observations for older ages than 100 years, especially in the best stands due to the shortening of rotation periods. It is considered particularly important to obtain data covering the early development stages of the stand for realistic predictions because first thinning can be applied at early ages and will condition the further development of the stand. In practical applications, erroneous predictions of the early development of the stands may result in, for example, erroneous predictions in the implementation of the first thinning.

Permanent sample plots recorded repeatedly over time are usually used to evaluate growth and forest yield and are especially effective as sampling method to evaluate changes in forest conditions (Gadow *et al.*, 1999). The lack of independence of data coming from these plots derive of their longitudinal and hierarchical

structure and means that data can be correlated temporal and/or spatially what may have implications for statistical analysis based on ordinary least squares (Gregoire *et al.*, 1995). In this work we have take into account within plot error correlation due to the longitudinal data structure including an autoregressive error structure and cross-equation correlation due to the multivariate structure of the system using SUR which is a technique that takes into account correlation between residuals of different models when estimating model parameters (Zellner, 1962). However, it is possible the existence of other types of correlation because of the hierarchical data structure. For example, it is possible the existence of spatial correlation between regions so sample plots that are within the same region are more likely to be similar to one another than sample plots that are in different regions.

The use of mixed models for evaluating such data has now become a common procedure in forestry research. Mixed models allow not only specify the within plot variance-covariance matrix but also consider between plot variation using mixed effects parameters. Estimates of parameters and their variances are obtained simultaneously based on a maximum likelihood procedure (Laird y Ware, 1982; Gregoire *et al.*, 1995). The simultaneous fit carried out in this study could involve the first step to fit mixed models that consider the hierarchical structure of the data. Instead of fitting the models with the assumption that the parameters are fixed, mixed effects models allow one or more parameters to vary by some hierarchical grouping variable.

Thinnings

The function to estimate the quadratic mean diameter after thinning is in Table 2, eq. 4. This paper assumes, in absence of thinning experimental networks to test the effect of thinnings on growth, that in the same site quality the growth per unit area of a thinning stand will be the same as an unthinning stand with the same basal area and age. This assumption may be correct given that thinnings have been light and moderate (Barrio-Anta *et al.*, 2006; Castedo-Dorado *et al.*, 2007). Clearly predictions may be inadequate for situations not represented in the data. For example, there may be some overestimation with heavy thinning, which for some time after the thinning, the growth is generally lower (García, 1990).

Mortality

In this equation (Table 2, eq. 5) reduction rate of the number of trees is proportional to initial number of trees (N_1), site index (SI) and an age function (t_i). In this equation the increase of site index for the same number of trees per hectare and same age, causes an increase in stand mortality. These results agree with those obtained in other studies (eg, Eid and Tuhus, 2001; Diéguez-Aranda *et al.*, 2005; Bravo-Oviedo *et al.*, 2006; Zhao *et al.*, 2006) and could be explained because in the most productive stands, with the other factors (spacing, genetics, age) being the same, stand dynamics is faster, faster individual growth can lead to greater intraspecific competition and therefore increase mortality. However, the effect of site index on mortality has not always produced the same results and therefore has not always been interpreted in this way. Some studies have found opposite effect of site index (Woolons (1998) or different effects depending on species and regions (Yao *et al.*, 2001; Yang *et al.*, 2003; Jutras *et al.*, 2003; Zhao *et al.*, 2007). This equation may not be necessary when there have been heavy thinnings because in such cases one might assume that there is no mortality due to competition.

Diameter distribution

The final equations are eqs. 6 and 7 in Table 2. We can explain the low R_{adj}^2 (41.78%) of the d_{min} equation because with different stands conditions we have small level of variability in this variable. The Kolmogorov-Smirnov goodness-of-fit test for the 324 distributions resulted in rejection of the null hypothesis at the 5% significance level for 35 diameter distributions, which means that approximately 11 percent of the real distributions could not be adequately modelled with the Weibull distribution. In a small number of cases where observed distributions differed from the predicted, visual inspection showed that observed distributions tended to be quite heterogeneous and this could be explained because *P. nigra* is an intermediate shade tolerant specie and understory trees are not always completely excluded. Once the parameters of the Weibull probability density function (p.d.f.) have been determined, the number of trees in each diameter class is obtained by integrating the p.d.f. between the class limits and multiplying by the number of trees per hectare in each diameter class. The average value of the EI was 212 (± 124) trees per hectare, error similar to those obtained in Cao (2004).

Table 2. Equations for predicting even-aged stands attributes of *P. nigra*

Equation ^a	
$\ln V_1 = 1.69 + 0.0604 \cdot SI - 48.8 \cdot t_1^{-1} + 0.982 \cdot \ln G_1$ $n = 271$; RMSE ^b = 26.6; $R_{adj}^2 = 96.99\%$; Mpr = -0.00000537; MApr = 18.10	[1]
$\ln G_2 = (t_1/t_2) \cdot \ln G_1 + 3.88 \cdot (1 - t_1/t_2) + 0.0475 \cdot (1 - t_1/t_2) \cdot SI$ $n = 271$; RMSE = 1.42; $R_{adj}^2 = 98.59\%$; Mpr = -0.000248; MApr = 1.04	[2]
$\ln V_2 = 1.69 + 0.0604 \cdot SI - 48.8 \cdot t_2^{-1} + 0.982 \cdot (t_1/t_2) \cdot \ln G_1 + 3.81 \cdot (1 - t_1/t_2) + 0.0466 \cdot (1 - t_1/t_2)$ $n = 271$; RMSE = 29.02; $R_{adj}^2 = 97.27\%$; Mpr = 0.00000479; MApr = 21.44	[3]
$dg_2 = 1.35 + 0.979 \cdot dg_1 + 0.859 \cdot (G_r/G_1) \cdot dg_1$ $n = 102$; RMSE = 1.00; $R_{adj}^2 = 98.53\%$; Mpr = -0.0064; MApr = 0.717	[4]
$N_2 = [N_1^{0.417} - 0.00003 \cdot SI \cdot (t_2^{1.57} - t_1^{1.57})]^{1/0.417}$ $n = 222$; RMSE = 11; $R_{adj}^2 = 99.94\%$; Mpr = -0.114; MApr = 8.10	[5]
$\bar{d} = dg - ((1/1 + \exp(0.021 \cdot SI + 0.00168 \cdot N + 0.0595 \cdot dg)) \cdot dg)$ $n = 324$; RMSE = 0.5; $R_{adj}^2 = 99.49\%$; Mpr = -0.069; MApr = 0.4	[6]
$d_{min} = \bar{d} - ((1/1 + \exp(-0.0288 \cdot dg + 0.0334 \cdot SI)) \cdot \bar{d})$ $n = 324$; RMSE = 3.5; $R_{adj}^2 = 41.78\%$; Mpr = 0.0948; MApr = 2.52	[7]

^a Notation: t : stand age in years; G : basal area in m^2 per hectare; G_r : basal area removed in the thinning in m^2 per hectare; \bar{d} : average diameter in cm; dg : quadratic mean diameter in cm; SI : site index in meters; N : number of trees per hectare; d_{min} : minimum diameter in cm. Subscripts 1 and 2 refer to time of measurement. ^b Statistics: root mean square error, $RMSE = \sqrt{\sum (y_i - \hat{y}_i)^2 / (n - p)}$; adjusted coefficient of determination, $R_{adj}^2 = 1 - [(n - 1) \cdot \sum (y_i - \hat{y}_i)^2 / (n - p) \cdot \sum (y_i - \bar{y}_i)^2]$; Mpr: mean PRESS residuals; MApr: mean absolute PRESS residuals where n : number of observations, p : number of parameters; y_i : observed value; \bar{y}_i : observed mean value; \hat{y}_i : predicted value.

Conclusion

This paper presents the results of fitting a dynamic model to estimate stand growth and production of even-aged stands of *P. nigra* in Spain, using variables typically measured in forest inventories. Due to the existence of correlation between the system equations and autocorrelation within each plot, volume and basal area equations were adjusted simultaneously using SUR with an autoregressive error structure. Independently, an equation was fitted to estimate the reduction in the number of trees due to regular mortality and an equation to determine the quadratic mean diameter after thinning. A parameter recovery framework was developed for its inclusion in the stand model to describe the diameter distributions of the stands using the stand level values of mean and minimum diameter. The resulting system of two nonlinear equations, which form the parameter recovery model, allowed for the determination of the scale and shape parameter.

The fitted equations allow us to estimate the number of trees, basal area and volume per unit area. The practical usability of the model derives from its greater flexibility compared to yield tables, unique models for *P. nigra* in areas where were the permanent plots used in this work. Different thinnings schedules, initial densities and rotation periods can be examined, allowing for an economic evaluation of different silvicultural alternatives in even-aged stands. From here, it could be established new relationships, such as generalized height-diameter and taper equations, which would provide additional information in forest management.

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